A Cayley graphs for Symmtric group on Degree four

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Abstract

In this paper, we determine all of subgroups of symmetric group S_4. First, we observe the multiplication table of S_4, then we determine all possibilities of every subgroup of order n, with n is the factor of order S_4. We found 30 subgroups of S_4. The diagram of Cayley graphs of S_4 is then presented.

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Keywords

Perumutation - symmetric Group - Cayley graph.

1 Introduction

For an arbitrary nonempty set S, define $A(S)$ to be the set of all one-to-one mapping of the set S onto itself. The set $A(S)$ with composition function operation is a group.If the set S contains n elements, then group $A(S)$ are denoted by S_n . Group S_n has $n!$ elements and will be called the symmetric group.There are many references on subgroups of S_2 and S_3. In this paper, we determine all subgroups of S_4 and then draw diagram of Cayley graphs of S_4 .

The number of subgroup of cyclic groups of order p^n where p is a prime number and this subgroups are finite cyclic groups. The subgroups of non abelian symmetric groups are S_2, S_3, S_4 and etc.

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Therefore,the result of this paper, that is a diagram of cayley graphs of S_4 is very important to determine the number of subgroup of S_4 .

2 Preliminary

Definition 2.1

A nonempty subset H of a group G is said to be a subgroupof G if, under the product in G, H itself forms a group.

Theorem 2.2

If G is a finite group and H is a sub-group of G, then order of H is a divisor of order G.

Theorem 2.3

If G is a finite group and $a \in G$, then order of a is a divisor of order G.

Theorem 2.4

Let G be a finite group and let $|G| = p^n m$ where $n \geq 1, p$ is a prime number and $(p, m) = 1$. Then G contains a subgroup of order p^i for each i where $1 \leq i \leq n$.

Definition 2.5

Let G be a finite group and let $|G| = p^n m$ where $n \geq 1, p$ is a prime number and $(p, m) = 1$. The subgroup of G of order p^n is called the sylow p subgroup of G.

Theorem 2.6

Let G be a finite group and let $|G| = p^n m$ where $n \geq 1, p$ is a prime number and $(p, m) = 1$. Then the number of Sylow p subgroup is of the form $(1 + kp)$, where k is a non-negative integer, and $(1 + kp)$ divides the order of G.

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Definition 2.7

A subgroup N of G is said to be a normal subgroup of G if for every $g \in G$ and $n \in N$, $gng^{-1} \in N$.

Theorem 2.8

There is a unique Sylow p -subgroup of the finite group G if only if it is normal.

Theorem 2.9

Let G be a group of order pq, where p and q are distinct primes and $p < q$. Then G has only one subgroup of order q. This subgroup of order q is normal in G.

3 Elements of Symmtric group

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Let $A = \{1, 2, 3, 4\}$ Then S_4 consists of

$$e = \begin{pmatrix}1&2&3&4\\1&2&3&4\end{pmatrix},\ P_{01} = \begin{pmatrix}1&2&3&4\\1&2&4&3\end{pmatrix},\ P_{02} = \begin{pmatrix}1&2&3&4\\1&3&4&2\end{pmatrix},\ P_{03} = \begin{pmatrix}1&2&3&4\\1&3&2&4\end{pmatrix},$$

$$P_{04} = \begin{pmatrix}1&2&3&4\\1&4&2&3\end{pmatrix},\ P_{05} = \begin{pmatrix}1&2&3&4\\1&4&3&2\end{pmatrix},\ P_{06} = \begin{pmatrix}1&2&3&4\\2&1&3&4\end{pmatrix},\ P_{07} = \begin{pmatrix}1&2&3&4\\2&1&4&3\end{pmatrix},$$

$$P_{08} = \begin{pmatrix}1&2&3&4\\3&1&4&2\end{pmatrix},\ P_{09} = \begin{pmatrix}1&2&3&4\\3&1&2&4\end{pmatrix},\ P_{10} = \begin{pmatrix}1&2&3&4\\4&1&2&3\end{pmatrix},\ P_{11} = \begin{pmatrix}1&2&3&4\\4&1&3&2\end{pmatrix},$$

$$P_{12} = \begin{pmatrix}1&2&3&4\\2&3&1&4\end{pmatrix},\ P_{13} = \begin{pmatrix}1&2&3&4\\2&4&1&2\end{pmatrix},\ P_{14} = \begin{pmatrix}1&2&3&4\\3&4&1&2\end{pmatrix},\ P_{15} = \begin{pmatrix}1&2&3&4\\3&2&1&4\end{pmatrix},$$

$$P_{16} = \begin{pmatrix}1&2&3&4\\4&2&1&3\end{pmatrix},\ P_{17} = \begin{pmatrix}1&2&3&4\\4&3&1&2\end{pmatrix},\ P_{18} = \begin{pmatrix}1&2&3&4\\2&4&3&1\end{pmatrix},\ P_{19} = \begin{pmatrix}1&2&3&4\\2&3&4&1\end{pmatrix},$$

$$P_{20} = \begin{pmatrix}1&2&3&4\\3&2&4&1\end{pmatrix},\ P_{21} = \begin{pmatrix}1&2&3&4\\3&4&2&1\end{pmatrix},\ P_{22} = \begin{pmatrix}1&2&3&4\\4&3&2&1\end{pmatrix},\ P_{23} = \begin{pmatrix}1&2&3&4\\4&2&3&1\end{pmatrix}.$$

In this group e is the identity element.
Thus S_4 is a group containing $4! = 24$ elements.

4 Cayley Table

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$\circ$	e	P_{01}	P_{02}	P_{03}	P_{04}	P_{05}	P_{06}	P_{07}	P_{08}	P_{09}	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	P_{20}	P_{21}	P_{22}	P_{23}
e	e	P_{01}	P_{02}	P_{03}	P_{04}	P_{05}	P_{06}	P_{07}	P_{08}	P_{09}	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	P_{20}	P_{21}	P_{22}	P_{23}
P_{01}	P_{01}	e	P_{03}	P_{02}	P_{05}	P_{04}	P_{07}	P_{06}	P_{09}	P_{08}	P_{11}	P_{10}	P_{19}	P_{18}	P_{21}	P_{20}	P_{23}	P_{22}	P_{13}	P_{12}	P_{15}	P_{14}	P_{17}	P_{16}
P_{02}	P_{02}	P_{05}	P_{04}	P_{01}	e	P_{03}	P_{19}	P_{18}	P_{21}	P_{20}	P_{23}	P_{22}	P_{07}	P_{06}	P_{09}	P_{08}	P_{11}	P_{10}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}
P_{03}	P_{03}	P_{04}	P_{05}	e	P_{01}	P_{02}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{06}	P_{07}	P_{08}	P_{09}	P_{10}	P_{11}	P_{19}	P_{18}	P_{21}	P_{20}	P_{23}	P_{22}
P_{04}	P_{04}	P_{03}	e	P_{05}	P_{02}	P_{01}	P_{13}	P_{12}	P_{15}	P_{14}	P_{17}	P_{16}	P_{18}	P_{19}	P_{20}	P_{21}	P_{22}	P_{23}	P_{07}	P_{06}	P_{09}	P_{08}	P_{11}	P_{10}
P_{05}	P_{05}	P_{02}	P_{01}	P_{04}	P_{03}	e	P_{18}	P_{19}	P_{20}	P_{21}	P_{22}	P_{23}	P_{13}	P_{12}	P_{15}	P_{14}	P_{17}	P_{16}	P_{06}	P_{07}	P_{08}	P_{09}	P_{10}	P_{11}
P_{06}	P_{06}	P_{07}	P_{08}	P_{09}	P_{10}	P_{11}	e	P_{01}	P_{02}	P_{03}	P_{04}	P_{05}	P_{15}	P_{16}	P_{17}	P_{12}	P_{13}	P_{14}	P_{23}	P_{20}	P_{19}	P_{22}	P_{21}	P_{18}
P_{07}	P_{07}	P_{06}	P_{09}	P_{08}	P_{11}	P_{10}	P_{01}	e	P_{03}	P_{02}	P_{05}	P_{04}	P_{20}	P_{23}	P_{22}	P_{19}	P_{18}	P_{21}	P_{16}	P_{15}	P_{12}	P_{17}	P_{14}	P_{13}
P_{08}	P_{08}	P_{11}	P_{10}	P_{07}	P_{06}	P_{09}	P_{20}	P_{23}	P_{22}	P_{19}	P_{18}	P_{21}	P_{01}	e	P_{03}	P_{02}	P_{05}	P_{04}	P_{15}	P_{16}	P_{17}	P_{12}	P_{13}	P_{14}
P_{09}	P_{09}	P_{10}	P_{11}	P_{06}	P_{07}	P_{08}	P_{15}	P_{16}	P_{17}	P_{12}	P_{13}	P_{14}	e	P_{01}	P_{02}	P_{03}	P_{04}	P_{05}	P_{20}	P_{23}	P_{22}	P_{19}	P_{18}	P_{21}
P_{10}	P_{10}	P_{09}	P_{06}	P_{11}	P_{08}	P_{07}	P_{16}	P_{15}	P_{12}	P_{17}	P_{14}	P_{13}	P_{23}	P_{20}	P_{19}	P_{22}	P_{21}	P_{18}	P_{01}	e	P_{03}	P_{02}	P_{05}	P_{04}
P_{11}	P_{11}	P_{08}	P_{07}	P_{10}	P_{09}	P_{06}	P_{23}	P_{20}	P_{19}	P_{22}	P_{21}	P_{18}	P_{16}	P_{15}	P_{12}	P_{17}	P_{14}	P_{13}	e	P_{01}	P_{02}	P_{03}	P_{04}	P_{05}
P_{12}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{03}	P_{04}	P_{05}	e	P_{01}	P_{02}	P_{09}	P_{10}	P_{11}	P_{06}	P_{07}	P_{08}	P_{22}	P_{21}	P_{18}	P_{23}	P_{20}	P_{19}
P_{13}	P_{13}	P_{12}	P_{15}	P_{14}	P_{17}	P_{16}	P_{04}	P_{03}	e	P_{05}	P_{02}	P_{01}	P_{21}	P_{22}	P_{23}	P_{18}	P_{19}	P_{20}	P_{10}	P_{09}	P_{06}	P_{11}	P_{08}	P_{07}
P_{14}	P_{14}	P_{17}	P_{16}	P_{13}	P_{12}	P_{15}	P_{21}	P_{22}	P_{23}	P_{18}	P_{19}	P_{20}	P_{04}	P_{03}	e	P_{05}	P_{02}	P_{01}	P_{09}	P_{10}	P_{11}	P_{06}	P_{07}	P_{08}
P_{15}	P_{15}	P_{16}	P_{17}	P_{12}	P_{13}	P_{14}	P_{09}	P_{10}	P_{11}	P_{06}	P_{07}	P_{08}	P_{03}	P_{04}	P_{05}	e	P_{01}	P_{02}	P_{21}	P_{22}	P_{23}	P_{18}	P_{19}	P_{20}
P_{16}	P_{16}	P_{15}	P_{12}	P_{17}	P_{14}	P_{13}	P_{10}	P_{09}	P_{06}	P_{11}	P_{08}	P_{07}	P_{22}	P_{21}	P_{18}	P_{23}	P_{20}	P_{19}	P_{04}	P_{03}	e	P_{05}	P_{02}	P_{01}
P_{17}	P_{17}	P_{14}	P_{13}	P_{16}	P_{15}	P_{12}	P_{22}	P_{21}	P_{18}	P_{23}	P_{20}	P_{19}	P_{10}	P_{09}	P_{06}	P_{11}	P_{08}	P_{07}	P_{03}	P_{04}	P_{05}	e	P_{01}	P_{02}
P_{18}	P_{18}	P_{19}	P_{20}	P_{21}	P_{22}	P_{23}	P_{05}	P_{02}	P_{01}	P_{04}	P_{03}	e	P_{14}	P_{17}	P_{16}	P_{13}	P_{12}	P_{15}	P_{11}	P_{08}	P_{07}	P_{10}	P_{09}	P_{06}
P_{19}	P_{19}	P_{18}	P_{21}	P_{20}	P_{23}	P_{22}	P_{02}	P_{05}	P_{04}	P_{01}	e	P_{03}	P_{08}	P_{11}	P_{10}	P_{07}	P_{06}	P_{09}	P_{17}	P_{14}	P_{13}	P_{16}	P_{15}	P_{12}
P_{20}	P_{20}	P_{23}	P_{22}	P_{19}	P_{18}	P_{21}	P_{08}	P_{11}	P_{10}	P_{07}	P_{06}	P_{09}	P_{02}	P_{05}	P_{04}	P_{01}	e	P_{03}	P_{14}	P_{17}	P_{16}	P_{13}	P_{12}	P_{15}
P_{21}	P_{21}	P_{22}	P_{23}	P_{18}	P_{19}	P_{20}	P_{14}	P_{17}	P_{16}	P_{13}	P_{12}	P_{15}	P_{05}	P_{02}	P_{01}	P_{04}	P_{03}	e	P_{08}	P_{11}	P_{10}	P_{07}	P_{06}	P_{09}
P_{22}	P_{22}	P_{21}	P_{18}	P_{23}	P_{20}	P_{19}	P_{17}	P_{14}	P_{13}	P_{16}	P_{15}	P_{12}	P_{11}	P_{08}	P_{07}	P_{10}	P_{09}	P_{06}	P_{02}	P_{05}	P_{04}	P_{01}	e	P_{03}
P_{23}	P_{23}	P_{20}	P_{19}	P_{22}	P_{21}	P_{18}	P_{11}	P_{08}	P_{07}	P_{10}	P_{09}	P_{06}	P_{17}	P_{14}	P_{13}	P_{16}	P_{15}	P_{12}	P_{05}	P_{02}	P_{01}	P_{04}	P_{01}	e

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5 Subgroups

According to the nontrivial subgroups of S_4 must have order 2, 4, 6, 8 or 12. We will determine all of the subgroups of S_4 . Clearly, the subgroup of S_4 of order 1 is the trivial subgroup $H_1 = \{e\}$.

Subgroups of order 2:

Let H be an arbitrary subgroup of S_4 of order 2. Since 2 is a prime number, then H is cyclic. Therefore H is generated by an element of S_4 of order 2. Thus, all subgroups of S_4 of order 2 are $H_2 = \{e, P_{01}\}$, $H_3 = \{e, P_{03}\}$, $H_4 = \{e, P_{05}\}$, $H_5 = \{e, P_{06}\}$, $H_6 = \{e, P_{07}\}$, $H_7 = \{e, P_{14}\}$, $H_8 = \{e, P_{15}\}$, $H_9 = \{e, P_{22}\}$, $H_{10} = \{e, P_{23}\}$.

Subgroups of order 3:

The subgroups of S_4 of order 3 is generated by an element of S_4 of order 3. Thus, all subgroups of S_4 of order 3 are $H_{11} = \{e, P_{02}, P_{04}\}$, $H_{12} = \{e, P_{08}, P_{13}\}$, $H_{13} = \{e, P_{09}, P_{12}\}$ $H_{14} = \{e, P_{10}, P_{19}\}$, $H_{15} = \{e, P_{11}, P_{18}\}$, $H_{16} = \{e, P_{16}, P_{20}\}$, $H_{17} = \{e, P_{17}, P_{21}\}$.

Subgroups of order 4:

Let H be an arbitrary subgroup of S_4 of order 4. then H is cyclic. Therefore H is generated by an element of S_4 of order 4. Thus,all subgroups of S_4 of order 4 are $H_{18} = \{e, P_{01}, P_{06}, P_{07}\}$, $H_{19} = \{e, P_{03}, P_{22}, P_{23}\}$, $H_{20} = \{e, P_{05}, P_{14}, P_{15}\}$, $H_{21} = \{e, P_{07}, P_{14}, P_{22}\}$, $H_{22} = \{e, P_{07}, P_{17}, P_{21}\}$, $H_{23} = \{e, P_{08}, P_{13}, P_{22}\}$, $H_{24} = \{e, P_{10}, P_{14}, P_{19}\}$.

Subgroups of order 6:

Let H be an arbitrary subgroup of S_4 of order 6. then H is cyclic. Therefore H is generated by an element of S_4 of order 6. Thus, all subgroups of S_4 of order 6 are $H_{25} = \{e, P_{01}, P_{02}, P_{03}, P_{04}, P_{05}\}$, $H_{26} = \{e, P_{01}, P_{15}, P_{16}, P_{20}, P_{23}\}$.

Subgroups of order 8:

Let H be an arbitrary subgroup of S_4 of order 8. then H is cyclic. Therefore H is generated by an element of S_4 of order 8. Thus, all subgroups of S_4 of order 8 are

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$H_{27} = \{e, P_{01}, P_{06}, P_{07}, P_{14}, P_{17}, P_{21}, P_{22}\}$, $H_{28} = \{e, P_{03}, P_{07}, P_{08}, P_{13}, P_{14}, P_{22}, P_{23}\}$,
$H_{29} = \{e, P_{05}, P_{07}, P_{10}, P_{14}, P_{15}, P_{19}, P_{22}\}$.

Subgroups of order 12:

Obviously the alternating group
$A_4 = H_{30} = \{e, P_{02}, P_{04}, P_{07}, P_{09}, P_{11}, P_{12}, P_{14}, P_{16}, P_{18}, P_{20}, P_{22}\}$. is a subgroup of S_4 of order 12. We will prove that A_4 is the unique subgroup of S_4 of order 12.
According to this result, we have the diagram of cayley graphs diagram is figure 1 below.

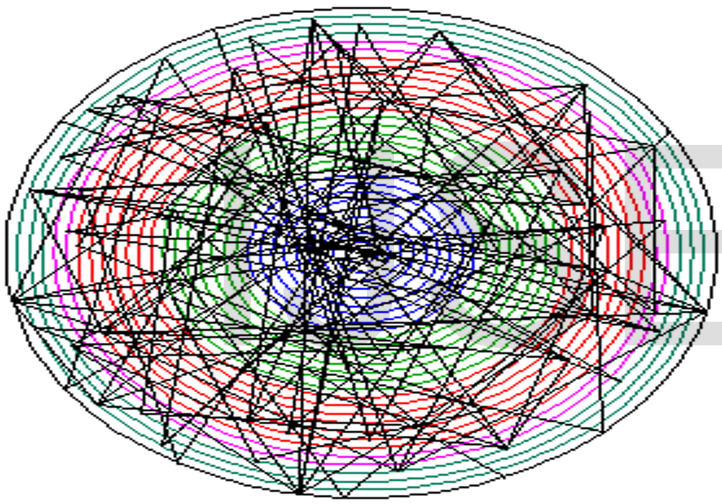

Figure 1: Cayley graphs of S_4

References

[1] **I.N.Herstein.** *"Topic in Algebra"*, (John Wiley and Sons,New York,1975).

[2] **J.B.Fraleigh.** *"A First Course in Abstract Algebra"*, (Addison-Wesley,London,1992).

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